

Combining Numbers and Units

Conversion Factors and how to use them

What is a conversion factor?

- It allows us to convert from one unit to another.
- It's a ratio.
- It is equivalent to the number 1.
- It does NOT change the size of the measurement, only the value associated with the unit.

Conversion factors come from a variety of situations:

Definitions within and between unit system give rise to conversion factors

Density can be, and usually is, used as a conversion factor

Concentration units are also used as conversion factors.

Percentages are often used as conversion factors

Conversion factors from within a unit system

- Come from the definitions within the system: i.e., 1cm = 10⁻² m, or 1 kg = 1000 g
- Are equivalent to the number 1
- Are derived from the division of one side by the other

$$\frac{1 \text{ cm}}{10^{-2} \text{ m}} \text{ or } \frac{10^{-2} \text{ m}}{1 \text{ cm}}$$

One thing to help you remember which number goes with which unit:

Always put the number "1" in front of the unit that has a prefix. The prefix is then replaced by what it means in front of the non-prefixed unit.

How are conversion factors used?

We use conversion factors by employing dimensional analysis.

What is dimensional analysis?

Dimensional analysis is a method of canceling out units so that the desired unit is the only one remaining.

Dimensional Analysis

- Determine the given values in the problem.
- Determine the quantity (unit) desired.
- State the question mathematically.
- Arrange the conversion factors so that all the units cancel except the desired unit.

How many centimeters are in 145.3 km?

- The given quantity here is 145.3 km
- The desired unit is centimeters.
- Only metric-to-metric conversion factors are needed.

$$? \text{ cm} = 145.3 \text{ km} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}} = 1.453 \times 10^7 \text{ cm}$$

Mathematical form of question Conversion to base unit Convert to desired unit

It is usually better to convert to the base unit first.
We can see that km will cancel.

We see now that m cancels.
Then we calculate.

Conversions between unit systems

- Conversions between unit systems are the same as within unit systems.
- These conversions are still based on multiplying by something that is equivalent to "1".
- We must use measured or defined conversion factors, instead of just using powers of 10 conversions.

Defined conversion factors between the metric and English systems

- 1 in = 2.540005 cm
- 1 qt = 0.946352946 L
- 1 lb = 453.5923744953 g

Temperature conversions

- Temperature conversions are different from other types of conversions because the zero is defined differently in different systems.
- In Celsius it is the freezing point of water, in Fahrenheit it's the temperature of a 50/50 mix of ice and salt, in Kelvin it's absolute zero.
- The difference in zero as well as the difference in unit size must be taken into account when converting.

Fahrenheit and Celsius

- Water freezes at 32°F
- Water freezes at 0°C
- Water boils at 212°F
- Water boils at 100°C
- Total range is 180°F
- Total range is 100°C

To convert we subtract the freezing point of water and multiply by the conversion factor:

$$\frac{180^\circ \text{F}}{100^\circ \text{C}} \text{ or } \frac{9^\circ \text{F}}{5^\circ \text{C}}$$

Kelvin and Celsius

- Water freezes at 273.15K
- Water freezes at 0°C
- Water boils at 373.15K
- Water boils at 100°C
- Total range is 100K
- Total range is 100°C

To convert we subtract the freezing point of water and multiply by the conversion factor:

$$\frac{100\text{K}}{100^\circ\text{C}} \text{ or } \frac{1\text{K}}{1^\circ\text{C}}$$

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Things to remember about temperature conversions

- All of the numbers in the equations are exact, except for the temperatures themselves.
- There is no degree symbol in the Kelvin unit. (K not °K)
- You must follow the sig fig rules for the equations.

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Temperature conversions

- Fahrenheit to Celsius

$$^\circ\text{C} = (^\circ\text{F} - 32^\circ\text{F}) \left(\frac{5^\circ\text{C}}{9^\circ\text{F}} \right)$$

- Celsius to Kelvin

$$\text{K} = (^\circ\text{C} - 0^\circ\text{C}) \left(\frac{1\text{K}}{1^\circ\text{C}} \right) + 273.15\text{K}$$

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Percentage conversions

- Percentage can be defined as:

$$\frac{\text{parts}}{\text{whole}} \times 100$$

- It can also be thought of as a conversion factor of so many parts out of 100 of the whole.

$$\frac{\text{parts}}{100 \text{ whole}}$$

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Like any other conversion factor, percentages can be used in either orientation.

$$\frac{100 \text{ whole}}{\text{parts}}$$

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Example Problem

- 14k Gold is 58.5% gold by mass. How many grams of gold are in 1.00 lb of 14k gold?

$$? \text{ g Au} = 1.00 \text{ lb 14k Au} \times \frac{453.59 \text{ g 14k}}{1 \text{ lb 14k}} \times \frac{58.5 \text{ g Au}}{100 \text{ g 14k}} = 265 \text{ g Au}$$

Mathematical form of question

Convert mass units (lb to g)

Use percentage to convert mass of 14 k to mass of Au

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Density

- Defined as mass divided by volume
- Can be used as a conversion factor
- States the equivalence between two different units for a substance

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Example problem

- The density of Aluminum is 2.70 g mL⁻¹. How many milliliters of aluminum has a mass of 1.50 kg?

$$? \text{ mL Al} = 1.50 \text{ kg Al} \times \frac{10^3 \text{ g Al}}{1 \text{ kg Al}} \times \frac{1 \text{ mL Al}}{2.70 \text{ g Al}} = 556 \text{ mL Al}$$

Mathematical form of the question Convert kg to g Use density to convert mass to volume

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Volume and Area

- Uses the same conversion factors as length
- The conversion factor is cubed for volume measurements
- The conversion factor is squared for area units

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Volume (cubic length units) example

$$\left(\frac{1 \text{ in}}{2.54 \text{ cm}} \right)^3$$

Area (square length units) example

$$\left(\frac{1 \text{ ft}}{12 \text{ in}} \right)^2$$

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Example problem

- A house has 1524 ft² of living space. What is this in m²?

$$? \text{ m}^2 = 1524 \text{ ft}^2 \times \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 \times \left(\frac{2.540 \text{ cm}}{1 \text{ in}} \right)^2 \times \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}} \right)^2 = 141.6 \text{ m}^2$$

Mathematical form of question Convert ft² to in² Convert in² to cm² Convert cm² to m²

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Example problem

- A fish tank measures 12 in by 20 in by 10 in. What is the volume of the tank in L?

$$? \text{ L} = (12 \text{ in} \times 20 \text{ in} \times 10 \text{ in}) \times \left(\frac{2.540 \text{ cm}}{1 \text{ in}} \right)^3 \times \frac{1 \text{ mL}}{1 \text{ cm}^3} \times \frac{10^{-3} \text{ L}}{1 \text{ mL}} = 39 \text{ L}$$

Mathematical form of question Convert in³ to cm³ Convert cm³ to mL Convert mL to L

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